MATH 202
Differential Equations
Dr. H. Yamani
Midterm
Summer 2018
Duration: 80 minutes

Problem	1	2	3	4	5	6	7	8	Total
Points	10	12	12	12	12	12	15	15	100
Scores									

Name:____________________________ **AUB ID:**__________________

Problem 1. (10 pts)
Find the general solution of the ODE:

$$y''' + 8y = 0$$

Problem 2. (12 pts)

Solve the following IVP.

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{y}{x} + x\cos^2\left(\dfrac{y}{x}\right) \\ y(1) = \dfrac{\pi}{4} \end{cases}$$

Problem 3. (12 pts)
Solve the following IVP.

$$\begin{cases} (x+2)\dfrac{dy}{dx} + y - (x+2)^2 y^5 = 0 \\ y(1) = 1 \end{cases}$$

Problem 4. (12 pts)
Multiply both sides of the ordinary differential equation by an appropriate factor to make it exact. Then solve the IVP.

$$\begin{cases} \left(2y^3 + 3xy^2\right)dx + \left(2x^2y + 3xy^2\right)dy = 0 \\ y(1) = 1 \end{cases}$$

Problem 5. (12 pts)
Show that the differential form in the following integral is exact, then evaluate the integral

$$\int_{(1,0,0)}^{(2,1,0)} \left(y^2 e^z + 2xe^z\right) dx + \left(2xye^z + \frac{2y}{y^2+1}\right) dy + \left(y^2 x e^z + x^2 e^z\right) dz$$

Problem 6. (12 points)
Let R be the region in the first quadrant bounded by the x-axis, the y-axis and the circle $x^2 + y^2 = 1$.
Let C be the boundary of R traced counterclockwise. Use Green's theorem to find the outward flux of the field $\mathbf{F} = \left(e^{-y} + 3x\right)i + \left(y + e^{-x} \cos x\right)\mathrm{j}$

Problem 7. (15 pts)
Use Stokes' theorem to evaluate the circulation of the field $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ around the boundary of the curve $x^2 + y^2 = 4$ in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$ traversed counterclockwise when viewed from above.

Problem 8 (15 pts)

Use the divergence theorem to find the outward flux of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ across the boundary of the region of the solid which is the entire surface of the upper cap cut from the solid sphere $x^2 + y^2 + z^2 \leq 25$ by the plane $z = 3$.